

Cosmic Acceleration Data and Bulk-Brane Energy Exchange

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Abstract

We consider a braneworld model with bulk-brane energy exchange. This allows for crossing of the $w = -1$ phantom divide line without introducing phantom energy with quantum instabilities. We use the latest SnIa data included in the Gold06 dataset to provide an estimate of the preferred parameter values of this braneworld model. We use three fitting approaches which provide best fit parameter values and hint towards a bulk energy component that behaves like relativistic matter which is propagating in the bulk and is moving at a speed v along the fifth dimension, while the bulk-brane energy exchange component corresponds to negative pressure and signifies energy flowing from the bulk into the brane. We find that the best fit effective equation of state parameter w_{eff} marginally crosses the phantom divide line $w = -1$. Thus, we have demonstrated both the ability of this class of braneworld models to provide crossing of the phantom divide and also that cosmological data hint towards *natural* values for the model parameters.

1 Introduction

Recent observations [1, 2] of distant type Ia supernova (SnIa) provide strong indication of a rather unconventional cosmic expansion profile for our universe at late times. It is suggested that we are currently in an accelerated expansion phase [1, 3], contrary to what we would expect in a matter dominated universe, where gravity tends to slow down the expansion. This result has had dramatic consequences on cosmological models, leading to the assumption of the existence of *dark energy*, an unknown component of the cosmic energy content which is responsible for the observed acceleration. Many models have been proposed to explain the nature of dark energy. These include new forms of matter and energy, like cosmological constant, scalar fields (quintessence), phantom fields, hessence, k-essence etc., as well as theories of modified gravity, extra dimensions and brane models [4, 5, 6, 7] (see [8, 9, 10, 11, 12] and [13] for a list of possible dark energy models). Apart from confirming the presence of dark energy, SnIa data also provide valuable information about its characteristics, namely the equation of state it obeys. Detailed

analysis of various available datasets implies that a redshift dependent equation of state parameter $w(z)$ crossing the phantom divide line $w = -1$ is consistent with the data [14] and may be favored by some datasets. If such crossing is verified, it will render a number of cosmological models as non-viable candidates for dark energy, as it happens with pure scalar field models of phantom or quintessence fields. Obtaining models which can account for both the late times accelerated expansion, as well as an equation of state parameter with $w = -1$ crossing becomes thus a high priority.

In [15] a model was presented based on brane cosmology, where dark energy is a manifestation of extra dimensions and the existence of bulk matter and energy exchange between the brane and the bulk. The model yields accelerated expansion at late times, as well as an effective equation of state parameter w_{eff} , which crosses the $w_{eff} = -1$ line. Modelling the bulk content as a slowly moving perfect fluid, we showed that the crossing can occur without invoking matter that violates the weak energy condition, provided that the dark radiation field enters with a negative sign contribution. This requirement can be bypassed if a more general ansatz is adopted. A similar but qualitatively different from a physical viewpoint situation occurs in models which consider a dark matter - dark energy interaction (see [16] for some examples). In [17] we departed from the bulk fluid interpretation and assumed that the bulk pressure and energy exchange contributions are the dominant ones, while the dark radiation field is negligible in the range $0 \leq z \leq 1$. We thus derived constraints on the allowed range of parameters for the bulk pressure and energy exchange terms by demanding the model to conform to the general expected temporal behavior [11, 18]. Our goal in this paper is to perform a more detailed analysis of the allowed range of parameters by fitting the model directly to the Gold supernova Ia dataset and checking whether the best fit values indicate accelerated expansion and $w = -1$ crossing, instead of requiring such a behavior a priori. In doing so, we will also keep the dark radiation term and see whether there are solutions which yield a positive sign for it.

We should note here that the back-reaction of cosmological perturbations on large scales can mimic the effects of background accelerated expansion [19], thus altering the results of any analysis that focuses only on the background evolution. We have chosen not to include the effects of perturbations in our analysis for two reasons:

- The set of equations of cosmological perturbations are known to be non-closed on the brane [20], i.e. information about the complete bulk evolution and dynamics is necessary in order to obtain a solution. Since the model we are considering doesn't originate from an exact bulk solution, but rather from a reasonable ansatz for the bulk matter content, a perturbative treatment cannot be particularly helpful.
- It turns out that at late times, when the Hubble radius is much greater than the bulk radius of curvature, the results of perturbations appear to be equivalent to those obtained when treating a conventional 4D cosmology [21].

2 General Framework

We will briefly review the model presented in [15, 17] and we will recast its equations in a form which is more convenient for the SNIa fit procedure. The model consists of a four-dimensional brane imbedded inside a five-dimensional bulk space. Both the bulk and the brane are allowed to carry some energy content, while there may also be an energy exchange term between the two. Einstein's equations in this setup are

$$\mathcal{G}_{MN} \equiv R_{MN} - \frac{1}{2} G_{MN}R = \frac{1}{4M^3} T_{MN}. \quad (1)$$

where M is the 5D Planck mass. The energy-momentum tensor T_{MN} is

$$T_{MN} = T_{MN}^{(B)} + T_{MN}^{(b)} - G_{MN}\Lambda - g_{\mu\nu}\sigma\delta(y)\delta_M^\mu\delta_N^\nu, \quad (2)$$

The first and second term are the bulk and brane content respectively, while the third comes from the bulk cosmological constant and the fourth is the brane tension term. The metric ansatz we use to study the cosmology of a Friedmann-Robertson-Walker brane is

$$G_{MN} = \begin{pmatrix} -n^2(y, t) & 0 & 0 \\ 0 & a^2(y, t)\gamma_{ij} & 0 \\ 0 & 0 & b^2(y, t) \end{pmatrix}, \quad (3)$$

where γ_{ij} is the metric for the maximally symmetric 3-space on the brane. For a *bulk energy-momentum tensor* $T_{MN}^{(B)}$ we shall take the general matrix

$$T^{(B)}_N^M = \begin{pmatrix} -\rho_B & 0 & P_5 \\ 0 & P_B\delta_j^i & 0 \\ -\frac{n^2}{b^2}P_5 & 0 & \bar{P}_B \end{pmatrix}, \quad T_{MN}^{(B)} = \begin{pmatrix} \rho_B n^2 & 0 & -n^2 P_5 \\ 0 & P_B a^2 \gamma_{ij} & 0 \\ -n^2 P_5 & 0 & \bar{P}_B b^2 \end{pmatrix}. \quad (4)$$

Note the presence of the off-diagonal component $T_5^0 = P_5$ signifying the flow of energy towards (or from) the brane. Note also the anisotropic choice $\bar{P}_B \neq P_B$, in general. Similarly, for a *brane energy-momentum tensor* $T_{MN}^{(b)}$ we shall take the matrix

$$T^{(b)}_N^M = \frac{\delta(y)}{b} \begin{pmatrix} -\rho & 0 & 0 \\ 0 & p\delta_j^i & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_{MN}^{(b)} = \frac{\delta(y)}{b} \begin{pmatrix} \rho n^2 & 0 & 0 \\ 0 & p a^2 \gamma_{ij} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (5)$$

Following [15, 22, 23, 24], we can use the junction conditions at the brane to derive the effective Friedmann equation on the brane that determines the cosmological evolution in four dimensions. Then, using an appropriate ansatz for the bulk pressure and the energy exchange term and assuming that the brane energy density is low, so we can neglect quadratic terms in ρ , we can exactly solve the system of equations to find the Hubble parameter. The ansatz we impose for the bulk energy-momentum tensor components which enter the cosmological equations on the brane is

$$\bar{P}_B = D a^\nu, \quad P_5 = F \left(\frac{\dot{a}}{a} \right) a^\mu. \quad (6)$$

and it can be physically justified once we model the bulk content as a relativistic fluid, slowly moving along the fifth dimension. The case $\mu = \nu$ corresponds to a slowly moving fluid with equation of state parameter $\omega = -1 - \mu/3$. Note that more than one "fluid" component may be present. In such a case, $\bar{P}_B = \sum_j D_j a^{\nu_j}$ and $P_5 = \frac{\dot{a}}{a} \sum_j F_j a^{\mu_j}$. Then, it is possible that $D_1 \gg D_i$, while $F_1 \ll F_i$ ($i \neq 1$) and therefore, at late times $\bar{P}_B \approx D_1 a^{\nu_1}$ and $P_5 \approx F_2 \left(\frac{\dot{a}}{a}\right) a^{\mu_2}$, accounting for different exponents.

Substituting these expressions, we arrive at the effective Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3} G_N \rho_{eff}, \quad (7)$$

where $G_N = 3\gamma/4\pi = 3\sigma/4\pi(24M^3)^2$ is the 4D Newton's constant and the *effective energy density* ρ_{eff} stands for

$$\rho_{eff} = \frac{\tilde{\mathcal{C}}}{a^{3(1+w)}} + \frac{\mathcal{C}/2\gamma}{a^4} - \frac{2\delta}{\gamma(\nu+4)} a^\nu + \frac{2(3w-1)F}{(\mu+4)[3(1+w)+\mu]} a^\mu. \quad (8)$$

where $\gamma \equiv \sigma(24M^3)^{-2}$ and we have defined $\delta \equiv D/24M^3$. We have assumed the equation of state $p = w\rho$ for matter on the brane. The constant $\tilde{\mathcal{C}}$ is just an integration constant for the energy density on the brane $\rho(a)$. The integration constant \mathcal{C} multiplies the *dark radiation* term and is related to global properties of the the bulk. The third and the fourth term in the expression for ρ_{eff} come from the bulk pressure \bar{P}_B and the brane-bulk energy exchange term P_5 respectively. The values of the parameters \mathcal{C} , D , F , ν and μ , related to the dark radiation term and the bulk contributions are crucial in determining the profile of cosmic expansion.

In [17] we performed an analysis for the allowed range of values for the latter four parameters, requiring a cosmological evolution that conforms to the general profile suggested by observational data, based on the assumption that the dark radiation term is negligible at late times. Here we shall perform a more general analysis, keeping also the dark radiation contribution and performing a fit to the Gold dataset, thus obtaining best fit values for the five parameters. In order to simplify the procedure, we will rewrite the effective Friedmann equation in terms of energy density parameters, assuming also a flat space ($k = 0$). It should be noted that actually there is an interesting degeneracy between spatial curvature and time-varying dark energy as pointed out in Refs [25, 26]. Therefore a small spatial curvature term could wipe out the mild trend for $w=-1$ crossing indicated by the Gold06 data in the context of a flat Universe. However, we have chosen not to allow for a possibility of non-zero spatial curvature for two reasons.

- There is significant theoretical motivation coming from inflation that $k=0$ to a very high accuracy. The possibility of non-zero spatial curvature would practically invalidate the flatness problem resolution provided by inflation.
- Allowing for of non-zero spatial curvature would introduce one more parameter to be determined by the SNIa data. However, the number of parameters involved in our brane world model is already large and this implies a large error region (see Fig. 1). The introduction of one more parameter would further extend this region making the results of our analysis very hard to interpret.

Doing so we obtain the following expression

$$\frac{H^2}{H_0^2} = \Omega_b a^{-3(1+w)} + \Omega_{DR} a^{-4} + \Omega_B a^\nu + \Omega_5 a^\mu, \quad (9)$$

where we have defined the energy density parameters for brane matter, dark radiation and bulk components as

$$\Omega_m (\tilde{C}) \equiv \frac{8\pi G_N}{3H_0^2} \tilde{C}, \quad \Omega_{DR} (C) \equiv \frac{C}{H_0^2}, \quad (10)$$

$$\Omega_B (D, \nu) \equiv -\frac{4D}{24M^3 H_0^2 (\nu + 4)}, \quad (11)$$

and

$$\Omega_5 (F, \mu) \equiv \frac{8\pi G_N}{3H_0^2} \frac{2(3w - 1)F}{(3(1+w) + \mu)(\mu + 4)}. \quad (12)$$

The brane energy density parameter Ω_m corresponds to the observed matter density. It has a value of approximately $\Omega_m \simeq 0.27$ [27]. Since the brane content is matter dominated, we will assume a value of $w = 0$ in the above expressions. The four density parameters are in fact related by the flatness requirement, which yields

$$\Omega_5 = 1 - \Omega_m - \Omega_{DR} - \Omega_B. \quad (13)$$

Thus, we can eliminate the Ω_5 term in favor of Ω_B . We see that it depends on both the coefficient D , which characterizes the bulk pressure, as well as the power ν . In order to make the dependence on ν explicit, we could rewrite it as $\Omega_B = \frac{\tilde{\Omega}_B}{\nu+4}$.

Using the above parametrization, we are now in position to fit the model to the supernova Ia data. There are four parameters to fit, Ω_{DR} , $\tilde{\Omega}_B$, ν and μ . Having determined the best fit values for these parameter, we can infer the temporal behavior of the deceleration parameter q , as well as the profile of the effective equation of state parameter w_{eff} . We refer to it as an effective parameter, since it isn't directly related to some particular form of matter, but is representative of the accumulated effects of both bulk matter and energy exchange, together with dark radiation due to the presence of extra dimensions. The parameter w_{eff} is according to the prescription of [28]

$$w_{eff}^{(D)} = -1 - \frac{1}{3} \frac{d \ln(\delta H^2)}{d \ln a}, \quad (14)$$

where $\delta H^2 = H^2/H_0^2 - \Omega_m a^{-3}$ accounts for all terms in the Friedmann equation not related to the brane matter. Substituting the above parametrization, we obtain the formula

$$w_{eff} (a) = -1 - \frac{1}{3} \left(\frac{-4\Omega_{DR} + \nu \Omega_B a^{\nu+4} + \mu \Omega_5 a^{\mu+4}}{\Omega_{DR} + \Omega_B a^{\nu+4} + \Omega_5 a^{\mu+4}} \right), \quad (15)$$

from which we can determine the variation of w_{eff} with time. As we see, all time dependence comes for the bulk terms. The *deceleration parameter* q is given by

$$q \equiv -\frac{1}{H^2 a} \frac{\ddot{a}}{a} = \frac{\Omega_m a^{-3} + 2\Omega_{DR} a^{-4} - (\nu + 2) \Omega_B a^\nu - (\mu + 2) \Omega_5 a^\mu}{2(\Omega_m a^{-3} + \Omega_{DR} a^{-4} + \Omega_B a^\nu + \Omega_5 a^\mu)}. \quad (16)$$

In the following treatment, we will re-express q and w_{eff} in terms of the redshift z , instead of the scale factor a . The two are related by $a = \frac{a_0}{1+z}$, a_0 being the current scale factor for our four-dimensional universe.

3 Gold dataset fit

We are going to use the updated Gold dataset (Gold06) compiled by Riess et al. [1], consisting of a total of 182 SNIa, located at distances ranging within the interval $0.024 \leq z \leq 1.755$. This dataset allows a fairly accurate reconstruction of the expansion history of our universe at late times.

The Gold dataset is compiled from various sources analyzed in a consistent and robust manner with reduced calibration errors arising from systematics. It contains 119 points from previously published data plus 16 points with $0.46 < z < 1.39$ discovered recently by the Hubble Space Telescope (HST). It also incorporates 47 points ($0.25 < z < 1$) from the first year release of the SNLS dataset [3] out of a total of 73 distant SNIa. Some supernovae were excluded [1] due to highly uncertain color measurements or high extinction $A_V > 0.5$, i.e. the dimming of the SN magnitudes resulting from dust. A redshift cut was also imposed at $cz < 7000 \text{ km/s}$ or $z < 0.0233$, in order to avoid the influence of a possible local ‘‘Hubble Bubble’’, i.e. a region with a higher Hubble parameter value due to the existence of a large local void [29]. Thus, a high-confidence subsample was defined.

The above observations provide the apparent magnitude $m(z)$ of the supernovae at peak brightness after implementing the correction for galactic extinction, the K-correction and the light curve width-luminosity correction. The resulting apparent magnitude $m(z)$ is related to the luminosity distance $d_L(z)$ through

$$m_{th}(z) = \bar{M}(M, H_0) + 5\log_{10}(D_L(z)), \quad (17)$$

where in a flat cosmological model

$$D_L(z) = (1+z) \int_0^z dz' \frac{H_0}{H(z'; a_1, \dots, a_n)}, \quad (18)$$

is the Hubble free luminosity distance ($H_0 d_L/c$). The parameters a_1, \dots, a_n are theoretical model parameters and \bar{M} is the magnitude zero point offset, depending on the absolute magnitude M and on the present Hubble parameter H_0 as

$$\begin{aligned} \bar{M} &= M + 5\log_{10}\left(\frac{c H_0^{-1}}{\text{Mpc}}\right) + 25 = \\ &= M - 5\log_{10}h + 42.38. \end{aligned} \quad (19)$$

The parameter M is the absolute magnitude and is assumed to be constant after the above mentioned corrections have been implemented in $m(z)$.

The data points of the Gold06 dataset, after the corrections have been implemented, are given in terms of the distance modulus as

$$\mu_{obs}(z_i) \equiv m_{obs}(z_i) - M. \quad (20)$$

The theoretical model parameters are determined by minimizing the quantity

$$\chi^2(a_1, \dots, a_n) = \sum_{i=1}^N \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_{\mu}^2 i + \sigma_v^2 i}, \quad (21)$$

where $\sigma_{\mu i}^2$ and $\sigma_{v i}^2$ are the errors due to flux uncertainties and peculiar velocity dispersion respectively. These errors are assumed to be gaussian and uncorrelated. The theoretical distance modulus is defined as

$$\mu_{th}(z_i) \equiv m_{th}(z_i) - M = 5\log_{10}(D_L(z)) + \mu_0, \quad (22)$$

where

$$\mu_0 = 42.38 - 5\log_{10}h, \quad (23)$$

$\mu_{th}(z_i)$ depends also on the parameters a_1, \dots, a_n used in the parametrization of $H(z)$ in equation (18).

To find the best fit parameters of the parametrization (9) we used the prior $\Omega_{0m} = 0.27$ and followed three approaches which we label as *minimization*, *marginalization-minimization* and *marginalization-averaging* with results which are consistent among the three approaches. These approaches can be described as follows:

- **Minimization:** We minimized eq. (21) with respect to all four parameters (Ω_{DR} , $\tilde{\Omega}_B$, ν and μ). The steps we followed for the minimization of (21) are described in detail in Ref. [30]. The advantage of this method is that it treats all parameters on an equal basis and gives direct information on all four of them. On the other hand it introduces significant degeneracy, thus leading to large error bars for the best fit parameter values.
- **Marginalization-Minimization:** In this approach we marginalize over $\Omega_{DR} \in [-1.5, 5]$ and $\tilde{\Omega}_B \in [-10, 2]$ and minimize with respect to ν and μ . To choose the range in which we will marginalize the two parameters, since we have no prior information for them, we minimized χ^2 for several pairs of reasonable values for ν and μ and found a wide enough range that contains all the best fit Ω_{DR} and $\tilde{\Omega}_B$. Also, we have checked that our results are not affected if we choose a larger range than the one we used to marginalize. The marginalized $\tilde{\chi}^2$ is defined as:

$$\tilde{\chi}^2(\nu, \mu) = -2 \ln \int e^{-\chi^2/2} d\Omega_{DR} d\tilde{\Omega}_B, \quad (24)$$

This is minimized with respect to ν and μ . We thus obtain best fit values for the exponents ν and μ which are directly relevant to the physical properties of the two components \bar{P}_B and P_5 . This approach has less degeneracy at the minimum compared to the previous direct minimization but it does not provide an estimate for Ω_{DR} and $\tilde{\Omega}_B$ which are treated as nuisance parameters. Such an estimate is needed in order to construct the best fit effective equation of state parameter $w_{eff}(z)$, defined by eq. (14). In order to obtain this estimate we fix ν and μ to the best fit values mentioned above and minimize χ^2 with respect to Ω_{DR} and $\tilde{\Omega}_B$.

- **Marginalization-Averaging:** The best fit values of Ω_{DR} and $\tilde{\Omega}_B$ obtained after marginalization as described above were also verified by considering the *average* values of these parameters instead of their values that minimize χ^2 . We thus define the average values of the two parameters as:

$$\langle \Omega_{par} \rangle = \frac{\int \Omega_{par} e^{-\chi^2/2} d\Omega_{DR} d\tilde{\Omega}_B}{\int e^{-\chi^2/2} d\Omega_{DR} d\tilde{\Omega}_B}, \quad (25)$$

where $par = \{\Omega_{DR}, \tilde{\Omega}_B\}$.

The best fit parameter values obtained with each one of the above methods are shown in Table I. The first approach cannot be used to draw a safe conclusion regarding the model under consideration due to the large error-bars of the best-fit parameters. On the other hand, the marginalization seems to hint towards $\nu \sim -3$ and a negative $\tilde{\Omega}_B$, which corresponds to a positive bulk pressure \bar{P}_B (see eqs. (6) and (10)). Also, a positive Ω_5 corresponds to $P_5 < 0$ which means that energy is flowing from the bulk into the brane. This is an intuitively reasonable result which may hint towards a relativistic matter component which is propagating in the bulk and is moving at a speed v along the fifth dimension [15, 17]. The dark radiation term has been shown to be related to the generalized comoving mass \mathcal{M} of the bulk fluid and its best-fit parameter Ω_{DR} is positive [31]. This is a desirable feature of this model, since a negative value would indicate a negative \mathcal{M} . It also ensures a well-behaved cosmological evolution at early times, although the validity of the effective Friedmann equation in this regime is questionable. Finally, as we can see in Table I, the two approaches, ‘Marginalization-Minimization’ and ‘Marginalization-Averaging’ are in good agreement.

The corresponding forms of the equation of state parameter $w_{eff}(z)$ and the deceleration parameter $q(z)$ are shown in Figs 1 and 2 for the first two approaches (‘Minimization’ and ‘Marginalization-Minimization’). Despite the large error-bars, the best-fit forms of $w_{eff}(z)$ and $q(z)$ are in good agreement between the two methods. The plots corresponding to the third approach (‘Marginalization-Averaging’) are practically identical with the Figs 1b and 2b of the ‘Marginalization-Minimization’ method (see also Table I).

A noteworthy feature of Figs 1a and 2a is the presence of “sweet-spots” at $z \sim 0.2$ and $z \sim 0.55$. The presence of “sweet-spots” at different redshifts for different parameterizations, usually polynomial, is something which has been studied previously (see [32]) and is a consequence of the ansatz used. However, upon marginalization the error region of Fig. 1b is significantly more smooth than the one of Fig. 1a.

Table 1: The best fit parameter values obtained with each one of the three methods. Note that the quantities that were determined with the *Averaging* procedure of eq. (25) were not assigned an error due to the nature of the method.

Method	ν	μ	Ω_{DR}	$\tilde{\Omega}_B$	Ω_5
Minimization	0.75 ± 5.09	-3.55 ± 5.77	0.29 ± 2.45	3.39 ± 0.76	-0.27 ± 2.57
Marg.-Min.	-3.0 ± 1.1	-0.8 ± 0.3	0.49 ± 0.25	-1.00 ± 0.49	1.24 ± 1.23
Marg.-Av.	-3.0 ± 1.1	-0.8 ± 0.3	0.52	-1.05	1.26

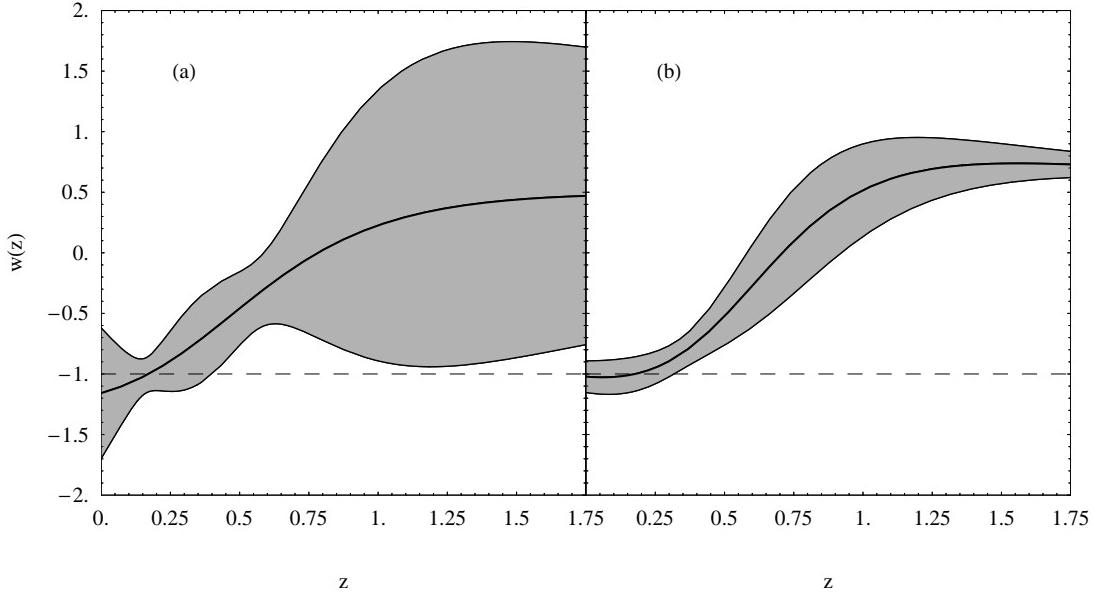


Figure 1: The effective equation of state w_{eff} for the ‘Minimization’ method (Fig. 1a) and the ‘Marginalization-Minimization’ approach (Fig. 1b). The large error region of the in Fig. 1a is due to the significant degeneracy of the model while the decreased error region in Fig. 1b is due to the fact that the values of ν and μ were assumed fixed during the minimization with respect to Ω_{DR} and $\tilde{\Omega}_B$.

4 Conclusion-Outlook

We have used the latest SnIa data included in the Gold06 dataset [2] to provide an estimate of the preferred parameter values of a braneworld model with bulk-brane energy exchange. The advantage of this model is that it has a clear physical motivation based on fundamental physics. Even though the large number of model parameters that require fitting introduces large error bars, the combination of different methods used for the fitting has provided interesting hints for the preferred parameter values.

Due to the significant degeneracy of the model used, the first fitting approach provided best fit parameter values with large error-bars (see Table I) and does not allow for safe conclusions to be drawn. On the other hand, the marginalization approach hints towards a bulk energy component that behaves like relativistic matter which is confined in the bulk and is moving at a speed v along the fifth dimension while the bulk-brane energy exchange component obtained corresponds to negative pressure, which means that energy is flowing from the bulk into the brane. Despite of these *natural values* of parameters, the best fit *effective* equation of state parameter w_{eff} marginally crosses the phantom divide line $w = -1$. It also reproduces accelerated expansion at late times, with a transition from acceleration to deceleration around $z \approx 0.5$, consistent with previous analysis [3, 1]. This demonstrates both the ability of this class of braneworld models to provide crossing of the phantom divide, while the cosmological data hint towards *natural*

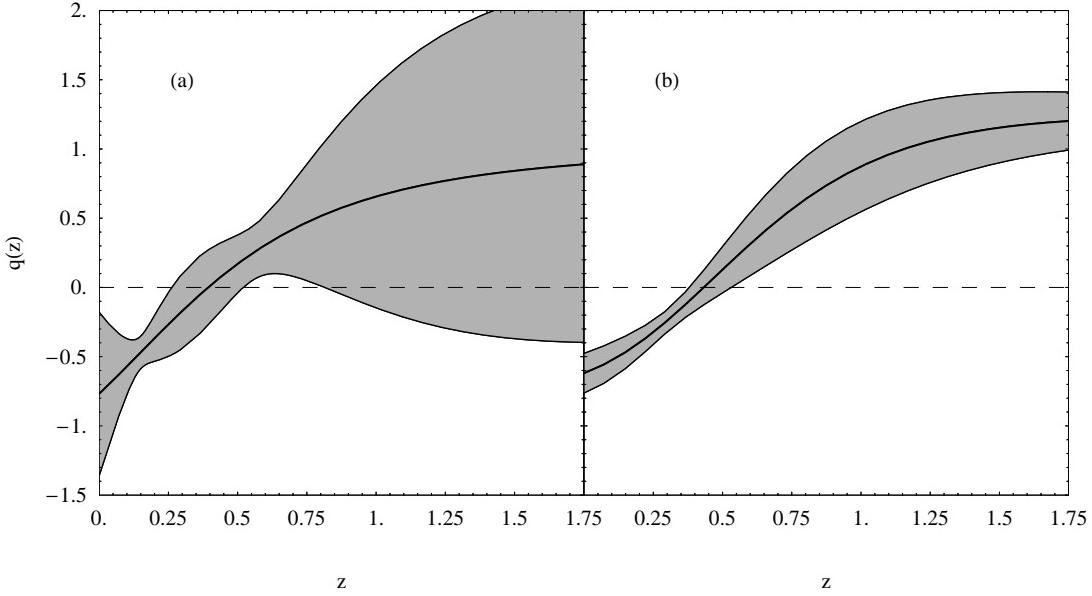


Figure 2: The deceleration parameter q for the ‘Minimization’ method (Fig. 2a) and the ‘Marginalization-Minimization’ approach (Fig. 2b). The large error region of the in Fig. 2a is due to the significant degeneracy of the model while the decreased error region in Fig. 2b is due to the fact that the values of ν and μ were assumed fixed during the minimization with respect to Ω_{DR} and $\tilde{\Omega}_B$.

values for the model parameters.

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